

# Solving Unit Commitment Problem Using Chemo-tactic PSO–DE Optimization Algorithm Combined with Lagrange Relaxation

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**Abstract**—This paper presents Chemo-tactic PSO-DE (CPSO-DE) optimization algorithm combined with Lagrange Relaxation method (LR) for solving Unit Commitment (UC) problem. The proposed approach employs Chemo-tactic PSO-DE algorithm for optimal settings of Lagrange multipliers. It provides high-quality performance and reaches global solution and is a hybrid heuristic algorithm based on Bacterial Foraging Optimization (BFO), Particle Swarm Optimization (PSO) and Differential Evolution (DE). The feasibility of the proposed method is demonstrated for 10-unit, 20-unit, and 40-unit systems respectively. The test results are compared with those obtained by Lagrangian relaxation (LR), genetic algorithm (GA), evolutionary programming (EP), and genetic algorithm based on unit characteristic classification (GAUC), enhanced adaptive Lagrangian relaxation (ELR), integer-coded genetic algorithm (ICGA) and hybrid particle swarm optimization (HPSO) in terms of solution quality. Simulation results show that the proposed method can provide a better solution.

**Index Terms**—Lagrangian Relaxation, Particle Swarm Optimization, Differential Evolution, Bacterial Foraging Optimization, Unit Commitment

## I. INTRODUCTION

Unit commitment (UC) is used to commit the generators such that the total production cost over the predicted load demand for scheduled time horizon is minimized considering the spinning reserve and operational constraints of generator units [1], [2]. Unit commitment is a high dimensional, nonlinear, non-convex, mixed-integer combinatorial optimization problem. Priority list method, integer programming, dynamic programming (DP), branch-and-bound methods, mixed-integer programming, and Lagrange relaxation (LR) are a few methods developed up to now for solving UC problem.

In priority list method, priority of units is determined from full load average production cost of the unit. The method is simple but the quality of solution is low. Priority list of units is also considered in Dynamic programming method. In spite of many advantages such as ability to maintain solution feasibility, for

higher dimension problem the problem size increases rapidly with the number of generators and this requires enormous computation time and large memory space. Branch-and-bound and mixed integer linear programming method also requires large computation time and memory space [3], [4]. Lagrange relaxation for UC problem is advanced than dynamic programming due to its faster computational time. The solution to Lagrange Relaxation for UC problem depends on the updating of Lagrange multipliers; hence it suffers from solution quality problem. This paper proposes a new hybrid heuristic method for solving UC problem. The proposed method is developed in such way that a Chemo-tactic PSO-DE optimization technique is applied to update Lagrange multipliers and this improves the performance of LR method. To illustrate the effectiveness of the proposed method, it is tested and compared to the conventional LR [5], GA [5], EP [6], GAUC [7], ELR [8], ICGA [9] and HPSO [10] for 10-unit, 20-unit, and 40-unit respectively.

In 2001, Prof. K. M. Passino proposed an optimization technique known as Bacterial Foraging Optimization Algorithm (BFOA) based on the foraging strategies of the E. Coli bacterium cells [11]. Until date there have been a few successful applications of the said algorithm in optimal control engineering, harmonic estimation [12], transmission loss reduction [13], machine learning [14] and so on. Experimentation with several benchmark functions reveal that BFOA possesses a poor convergence behavior over multimodal and rough fitness landscapes as compared to other naturally inspired optimization techniques like the Genetic Algorithm (GA), Particle Swarm Optimization (PSO) and Differential Evolution (DE). Its performance is also heavily affected with the growth of search space dimensionality. In 2007, Kim et al. proposed a hybrid approach involving GA and BFOA for function optimization [15]. The proposed algorithm outperformed both GA and BFOA over a few numerical benchmarks and a practical PID tuner design problem [16-19].

## II. NOMENCLATURE

$P_i^t$	: Generation output power of unit $i$ at hour $t$ .
$U_i^t$	: Status of unit $i$ at hour $t$ (on=1, off=0).
$ST_i^t$	: Startup cost of generator $i$ at hour $t$ .
$F_i(P_i^t)$	: Generator fuel cost function in quadratic form
$F_i(P_i^t) = a_i(P_i^t)^2 + b_iP_i^t + c_i$	(\$/h)
$F(P_i^t, U_i^t)$	: Total production cost.
$N$	: The number of generator units.
$N_S$	: Swimming length $N_s$ is the maximum number of steps taken by each bacterium when it moves from low nutrient area to high nutrient area.
$S$	: The number of bacteria in the population.
$N_C$	: The number of chemo-tactic steps.
$T$	: The number of hours.
$pbest$	: The best previous position of $k^{th}$ particles.
$gbest$	: The index of the best particle among all particles in the group.
$F$	: The scale factor
$CR$	: The Cross over probability
$C_1, C_2$	: Acceleration constants
$rand, Rand$	: Random value in the range [0, 1]
$P_{load}^t$	: Load demand at hour $t$ .
$R^t$	: Spinning reserve at hour $t$ .
$P_{i,min}$	: Minimum generation limit of generator $i$
$P_{i,max}$	: Maximum generation limit of generator $i$
$T_{i,up}$	: Minimum up time of generator $i$
$T_{i,down}$	: Minimum down time of generator $i$
$HST_i$	: The unit's hot startup cost.
$CST_i$	: The unit's cold startup cost.
$T_{i,cold}$	: The cold start hour.
$\lambda^t, \mu^t$	: Lagrange multipliers
$V\lambda^t(k)$	: Velocity vector of $\lambda^t$ in $t^{th}$ hour for $k^{th}$ bacterial population.
$V\mu^t(k)$	: Velocity vector of $\mu^t$ in $t^{th}$ hour for $k^{th}$ bacterial population.

## III. PROBLEM FORMULATION

From the definition of UC mentioned above, the objective of UC problem is to minimize the production cost over the schedule time horizon (e.g., 24h). The

objective function can be formulated mathematically as an optimization problem as follows:

$$F(P_i^t, U_i^t) = \sum_{t=1}^T \sum_{i=1}^N [F_i(P_i^t) + ST_i^t(1 - U_i^{t-1})]U_i^t \quad (1)$$

subject to

(a) Power balance constraint

$$P_{load}^t - \sum_{i=1}^N P_i^t U_i^t = 0 \quad (2)$$

(b) Spinning reserve constraint

$$P_{load}^t + R^t - \sum_{i=1}^N P_{i,max} U_i^t \leq 0 \quad (3)$$

(c) Generation limit constraints

$$P_{i,min} U_i^t \leq P_i^t \leq P_{i,max} U_i^t \quad (4)$$

(d) Minimum up and down time constraints

$$U_i^t = \begin{cases} 1 & \text{if } T_{i,on} < T_{i,up}, \\ 0 & \text{if } T_{i,off} < T_{i,down}, \\ 0 \text{ or } 1, & \text{otherwise,} \end{cases} \quad (5)$$

(e) Startup cost

$$ST_i^t = \begin{cases} HST_i, & \text{if } T_{i,down} \leq T_{i,off} \leq T_{i,cold} + T_{i,down} \\ CST_i, & \text{if } T_{i,off} > T_{i,cold} + T_{i,down} \end{cases} \quad (6)$$

## IV. OVERVIEW OF PARTICLE SWARM OPTIMIZATION

The Particle Swarm Optimization was introduced by James Kennedy and Russell C. Eberhart in 1995 [20]. It is similar to other evolutionary computational techniques those are GA and EP in that PSO initializes a population of individuals randomly. These individuals are known as particles and have positions and velocities. In addition, it searches for the optimum by updating generations, and population evolution is based on the previous generations. PSO is motivated from the simulation of the behavior of social systems such as fish schooling and birds flocking. In PSO, each particle flies through the problem space by following the current optimal particles. Each individual adjusts its flying according to its own flying experience and its neighbors flying experience. Based on its own thinking the particle attracts to its best position ( $pbest$ ) the particle changes its velocity based on the social-psychological adaptation of knowledge that is the particle attracts its previous best position among the group ( $gbest$ ), here so the velocity of the particle is updated to new position according to its modified velocity using the information.

- The current velocity
- The distance between the current position and  $pbest$ .
- The distance between the current position and  $gbest$ .

$$V_{ij}(iter+1) = w * V_{ij}(iter) + C_1 * rand_1 * (pbest_{ij} - x_{ij}(iter)) + C_2 * rand_2 * (gbest_i - x_{ij}(iter)) \quad (7)$$

where

$w$  = inertia weight factor

$x_{ij}(iter)$  = current position of  $i^{th}$  particle in  $j^{th}$  dimension at iteration (iter)

$V_{ij}(iter)$  = velocity of  $i^{th}$  particle in  $j^{th}$  dimension at iteration (iter)

Appropriate selection of inertia weight provides a balance between global and local explorations. The updated position of each particle is expressed as

$$x_{ij}(iter+1) = x_{ij}(iter) + V_{ij}(iter+1) \quad (8)$$

The new positions are calculated repeatedly till a pre-specified maximum number of iterations are reached.

## V. OVERVIEW OF DIFFERENTIAL EVOLUTION OPTIMIZATION

Differential Evolution (DE) is a new floating point encoded evolutionary algorithm for global optimization developed by Price and Storn in 1995 [21]. It is similar to other evolutionary computation technique and also starts with a population of NP. It is a D-dimensional search variable vector. A special kind of differential operator is used to create new offspring from parent chromosomes instead of classical crossover or mutation. In each generation to change each population member  $X_i$ , a Donor vector  $v_i$  is created. Three parameter vectors  $r1$ ,  $r2$ , and  $r3$  are chosen in a random fashion from the current population. A scalar number  $F$  scales the difference of any two of the three vectors and the scaled difference is added to the third one. Donor vector for  $j^{th}$  component of  $k^{th}$  population at  $iter$  generation is expressed as

$$v_{k,j}(iter+1) = X_{r1,j}(iter) + F * (X_{r2,j}(iter) - X_{r3,j}(iter)) \quad (9)$$

CR is called "Crossover" constant and it appears as a control parameter of DE. The crossover is performed on each of the D variables whenever a randomly picked number between 0 and 1 is within the CR value. The trial vector  $u_{k,j}$  is outlined as

$$u_{k,j}(iter) = v_{k,j}(iter) \quad \text{if } rand(0,1) < CR \\ u_{k,j}(iter) = X_{k,j}(iter) \quad \text{otherwise} \quad (10)$$

## VI. OVERVIEW OF CHEMOTACTIC PSO-DE

PSO and DE are excellent heuristics like other evolutionary algorithms. Practical experiences suggest that they reach stagnation after certain number of generations as the population is not converged locally, so they will stop proceeding towards global optimal

solutions. Chemo-tactic process of bacterial foraging describes the movement of an E.coli cell through swimming and tumbling via flagella. It can move in two different ways, it can swim for a definite period of time or it can tumble and it alternates between these two modes of operation for the entire lifetime. For example if  $\theta_k$  denotes the  $k^{th}$  bacterium and  $C_k$  is the size of the step taken in the random direction specified by the tumble. Then the movement of bacterium is represented by

$$\theta_k = \theta_k + C_k * Delta_k \\ Delta_k = \frac{\Delta_k}{\sqrt{\Delta_k^T * \Delta_k}} \quad (11)$$

where  $\Delta$  is a unit length vector in the random direction

In this article we come up with a hybrid optimization technique, which synergistically couples the BFOA with the PSO, DE and Lagrange multipliers method. The proposed algorithm performs local search through the chemo-tactic movement operation of BFOA whereas the global search over the entire search space is accomplished by a PSO-DE operator. In this way it balances between exploration and exploitation enjoying best of both the worlds.

## VII. METHODOLOGY

Since UC was introduced, several methods have been used to solve this problem. Among those methods, LR seems to be the most appropriate one [5], [8]. This method solves the UC problem by relaxing or temporarily ignoring the coupling constraints, power balance and spinning reserve requirements, then solving the problem through a dual optimization procedure. The dual procedure attempts to reach the constrained optimum by maximizing the Lagrange function with respect to the Lagrange multipliers.

$$L(P, U, \lambda, \mu) = F(P_i^t, U_i^t) + \sum_{i=1}^T \lambda \left( P_{load}^t - \sum_{i=1}^N P_i^t U_i^t \right) + \sum_{i=1}^T \mu^t \left( P_{load}^t + R^t - \sum_{i=1}^N P_{i,max} U_i^t \right) \quad (12)$$

while minimize with respect to the power output and generating unit status, that is

$$q^*(\lambda, \mu) = \max_{\lambda, \mu} q(\lambda, \mu) \quad (13)$$

where

$$q(\lambda, \mu) = \min_{P_i^t, U_i^t} L(P, U, \lambda, \mu) \quad (14)$$

Substitute (1) in (9), the Lagrangian function becomes

$$L = \sum_{i=1}^N \sum_{t=1}^T \left\{ [F_i(P_i^t) + ST_i^t (1 - U_i^{t-1})] U_i^t - \lambda P_i^t U_i^t - \mu^t P_{i,max} U_i^t \right\} + \sum_{t=1}^T (\lambda P_{load}^t + \mu^t (P_{load}^t + R^t)) \quad (15)$$

Equations (2) and (3) are the coupling constraints across the units. The coupling constraints in the above equation are temporarily ignored and then the minimum of Lagrangian function is solved for each generating unit, without regard for what is happening on the other generating units.

$$\min_{P_i^t, U_i^t} L(P, U, \lambda, \mu) = \sum_{i=1}^N \min \sum_{t=1}^T \left\{ [F_i(P_i^t) + ST_i^t(1 - U_i^{t-1})]U_i^t \right\} \quad (16)$$

Therefore the minimum for each generating unit over all time periods is:

$$\min q(\lambda, \mu) = \sum_{i=1}^N \min \sum_{t=1}^T \left\{ [F_i(P_i^t) + ST_i^t(1 - U_i^{t-1})]U_i^t \right\} \quad (17)$$

subject to constraints (4) and (5). This problem is solved through a two-state dynamic programming problem in two variables for each unit. On the other hand, in order to maximize the Lagrange function with respect to the Lagrange multipliers, the adjustment of Lagrange multipliers must be done carefully. Most of research works use sub-gradient method to achieve this task. In this paper, we use the Chemo-tactic PSO-DE optimization to adjust the Lagrange multipliers and improve the performance of Lagrange relaxation method.

This is solved as a dynamic programming problem in one variable with two possible unit states ( $U_i^t = 0$  or 1).

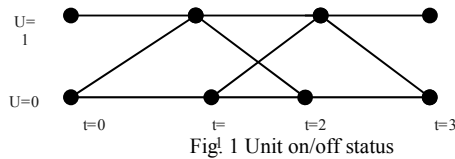


Fig1. Unit on/off status

The dynamic programming part must take into account all the start up costs as well as the minimum up and down time for the generating units.

At  $U_i^t=0$  state the value of function to minimize is zero. At  $U_i^t=1$  state the function to be minimized is  $\min[F_i(P_i^t) - \lambda^t P_i^t]$ . The minimum of this function is found by taking the first derivative

$$\frac{d}{dP_i^t} [F_i(P_i^t) - \lambda^t P_i^t] = 0 \quad (18)$$

The dual power  $P_i^{t,opt}$  is obtained from (19)

$$P_i^{t,opt} = \frac{\lambda^t - b_i}{2c_i} \quad (19)$$

$$\begin{aligned} \text{if } P_i^{t,opt} < P_{i,min}, \text{ then } P_i^t &= P_{i,min} \\ \text{if } P_i^{t,opt} > P_{i,max}, \text{ then } P_i^t &= P_{i,max} \end{aligned} \quad (20)$$

$$\text{if } P_{i,min} \leq P_i^{t,opt} \leq P_{i,max}, \text{ then } P_i^t = P_i^{t,opt}$$

The dual power obtained from (20) is substituted in on/off decision criterion DC

$$DC_i^t = [F_i(P_i^t) + ST_i^t(1 - U_i^{t-1}) - \lambda^t P_i^t - \mu^t P_{i,max}] \quad (21)$$

If  $DC_i^t \leq 0$  the unit will be committed if it does not violate minimum downtime constraint, otherwise the unit is decommitted if it does not violate the minimum uptime constraint. A step-by-step procedure for the proposed method is outlined as follows.

#### Section -A:

**Step1:** Compute  $\lambda^t$  and  $\mu^t$  by Chemo-tactic PSO-DE.

**Step2:** Set the unit status by calculating dual power and obtain on/off decision DC (21).

**Step3:** Calculate the spinning reserve for generated 'U' status at each hour of the population. If excessive spinning reserve is observed decommit the unit with high full load average production cost if it does not violate the minimum uptime constraint. Otherwise if the unit violates, the unit with next high full load average production cost is decommitted. Now set the spinning reserve according to the new status and repeat until the spinning reserve satisfies.

**Step4:** If less spinning reserve than required is observed commit the unit with less full load average cost if it does not violate the minimum downtime constraint. Otherwise if it violates, the unit with next less full load average production cost is committed. Now set the spinning reserve according to the new status and repeat until the spinning reserve satisfies.

**Step5:** Economic dispatch for the generated U-status is carried out by Lagrangian multiplier method.

#### Section-B:

**Step1:** Randomly generate  $\lambda^t, \mu^t, V\lambda^t$ , and  $V\mu^t$  with in the range for initialized bacterium. Generate unit-status U as in section-A and calculate  $pbest$  for U,  $\lambda^t, \mu^t$ , cost and fit (J and JF) and  $gbest$  for U,  $\lambda^t, \mu^t, J$  and JF.

**Step2:** Starting of the chemo-tactic loop (it=it+1)

**Step3:** For each bacteria (k=k+1)

$$JF_{last} = JF(k)$$

**Step4:** Calculate  $V\lambda^t, V\mu^t$  and check for maximum and minimum velocity limits.

$$V\lambda(k) = V\lambda(k)$$

$$+ C_1 * Rand * (pbest\lambda^t(k) - \lambda^t(k)) \quad (22)$$

$$+ C_2 * Rand * (gbest\lambda^t - \lambda^t(k))$$

$$V\mu(k) = V\mu(k)$$

$$+ C_1 * Rand * (pbest\mu^t(k) - \mu^t(k)) \quad (23)$$

$$+ C_2 * Rand * (gbest\mu^t - \mu^t(k))$$

**Step5:** Move to new position

$$\lambda(k) = \lambda(k) + C_\lambda * V\lambda(k) \quad (24)$$

$$\mu(k) = \mu(k) + C_\mu * V\mu(k)$$

where  $C_\lambda$  and  $C_\mu$  are chemo-tactic step size

**Step6:** Handle limits

$$\lambda_{min} \leq \lambda_k \leq \lambda_{max}$$

$$\mu_{min} \leq \mu_k \leq \mu_{max} \quad (25)$$

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## IX. CONCLUSION

Unit commitment problem is solved with a new methodology Chemo-tactic PSO-DE Optimization algorithm. PSO and DE are incorporated in chemo-tactic algorithm to update Lagrange multipliers so that they are suitable for high dimensional combinatorial optimization problem. Results show that as the dimension of the problem increases it generates good unit status with lower production cost. Hence it can be concluded that the proposed method provide lower production cost than those of LR, GA, EP, GAUC and ELR methods for 40-unit system and also the problem can be extended for 60-unit, 80-unit and 100-units as in this method better costs are obtained as the dimension (number of generators) of the problem increases.

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